

When Falling Stars hit the Zero Lower Bound: Implications for Dynamic Term Premia

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September 1, 2026

Abstract

Recent studies emphasize how including declining trends in nominal yields (falling stars) into Gaussian affine term structure models produces more reliable estimates of the term premium (TP). However, existing falling stars models typically do not account for structural changes that are associated with a binding zero lower bound (ZLB). To explore the effects these structural changes have on TP estimation beyond that implied by the falling stars model, we propose and estimate a novel arbitrage-free affine term structure model that integrates both falling stars and regime shifts between ZLB and non-ZLB states. Our findings reveal that neglecting these regime shifts may lead to underestimation of the term premium during non-ZLB states, and to an inability to capture temporary increases in TP volatility during ZLB periods. These results underscore the necessity of incorporating ZLB-induced structural instability alongside falling stars for comprehensive term premium estimation.

(JEL Classification: C11, C32, C52, E43)

Keywords: affine term structure model, market expectation, regime switching, Bayesian estimation

1 Introduction

The term premium (TP) is a central object of interest produced by term structure models. Kim and Wright (2005) define the TP as the compensation received by bondholders for shouldering the risk associated with holding a long-term treasury bond, which is directly linked to macroeconomic risk. Essentially, the TP measures the extent of macroeconomic uncertainty faced by investors. Estimation of the TP also enables us to decompose bond yields into their expectation and risk premium components. Bauer and Rudebusch (2014) argue that these components act as pathways through which the effects of macroeconomic policies, such as large-scale asset purchases (LSAP), are transmitted throughout the economy. Accurate estimation of the TP during ZLB periods is therefore also vital for understanding the effects of unconventional monetary policy.

Recent studies utilizing affine term structure models (ATSM), notably Bauer and Rudebusch (2020), reveal that the choice of whether to model the latent yield factors as stationary or non-stationary has significant implications for TP estimation. Cohen, Hördahl, and Xia (2018) note that term premia derived from models not accounting for interest rate trends exhibit a clear downward trend, suggesting that a substantial part of the variation in long-term yields is due to the TP. In contrast, models that include interest rate trends, which we refer to as falling stars models, indicate that the TP tends to be mean-reverting and attributes much of the variation in yields to fluctuations in the expectations component. This difference is more than just cosmetic; for instance, Bauer, Rudebusch, and Wu (2012) and Christensen and Rudebusch (2016) highlight how neglecting macroeconomic trends when modeling the yield curve leads to significant biases in the estimation of parameters and, by extension, the TP.

A separate strand of the literature has taken note of how the dynamics of short-term yields, collectively referred to as the short rate, undergo significant structural changes

when the zero lower bound (ZLB) is binding. Heavily emphasized is how the volatility of the short end of the yield curve decreases drastically under a binding ZLB. Christensen and Rudebusch (2015) show that this so-called compression of volatility appears as a natural consequence of shadow-rate modeling; Nakata and Tanaka (2016) furnish a fully structural DSGE model that accounts for the ZLB in which the same phenomenon arises. Such changes in the dynamics of short-term yields, in particular those pertaining to the volatility of the short rate, are likely to affect the level and volatility of the TP by altering estimates of expected excess bond returns.

We ask whether structural instability associated with the lower bound materially alters term premium estimates even after accounting for falling stars. To answer this question, we develop the FS-ZLB model, which combines falling interest-rate trends with regime-dependent dynamics across normal and lower-bound states. The answer is yes. The term premium estimates produced by the FS-ZLB model differ materially from both those obtained from classical Gaussian ATSMs and those obtained from a falling-stars model that does not allow for structural changes at the lower bound.

The inclusion of falling stars in the FS-ZLB model keeps the expected short rate close to zero, in contrast to the expected short rate in Gaussian ATSMs, which typically revert to a positive mean. This results in a lower expectations component and a higher TP in the FS-ZLB model compared to Gaussian ATSMs. We also find that the FS-ZLB estimate of the TP is more volatile during the lower bound regime compared to the estimate obtained from the FS-only model. Perhaps more surprisingly, allowing for regime shifts affects term-premium estimates even when the economy is in the normal regime.

In the FS-ZLB model, even while the economy is in the normal regime, the possibility of entering the lower-bound regime affects the expected future path of short rates, and the cumulative probability grows with the forecast horizon. A key feature behind this result is that the probability of entering the lower bound regime remains positive even when the

current short rate is well above the lower bound, so that future regime risk affects current long-horizon expectations. This leads to short-rate expectations in the FS-ZLB model that slowly decline with the forecast horizon. This is in contrast to our FS-only benchmark, where the unit-root specification implies that expected future short rates remain at the current short rate at every horizon. Intuitively, investors account for the possibility that the economy may enter a low-rate regime in the future, even when the economy is currently away from the lower bound. Such differences in how investors form short rate expectations across the two models explain why they produce different TP estimates even during the normal regime, and suggests that the knowledge of possible structural shifts affects TP estimation independently of the actual shifts themselves.

The greater volatility of the FS-ZLB term premium during lower-bound periods reflects a shift in the relative importance of the yield factors for the market price of risk. In the FS-ZLB model, the compression of the short rate at the lower bound reduces level factor volatility, while the slope and curvature factors become substantially more important in determining market prices of risk and by extension the TP. Empirically, the loadings of the slope and curvature factors on the TP sharply increase in the lower-bound regime relative to both the normal regime and the FS-only model. This pattern is consistent with the interpretation that, when short rates are constrained and therefore contain less information about future monetary policy, investors place relatively greater weight on information that can be recovered from the remainder of the yield curve. As a result, variation in the slope and curvature factors is transmitted more strongly into market prices of risk and term premia during the lower bound regime.

The preceding comparison of the TP across the FS-ZLB and FS-only models reveals that the TP plays a fundamentally different role as the main driver of yield variation during ZLB periods. More broadly, our results show that ZLB risk can matter for TP inference even outside realized lower bound episodes. Models that allow for falling interest-rate trends

but impose time-invariant conditional dynamics may therefore miss economically important variation in both expected short rates and term premia.

The workhorse FS-ZLB model used to investigate the sensitivity of TP estimation to ZLB-related structural instability is part of the class of arbitrage-free Nelson–Siegel (AFNS) models. Our model includes a unit root within the factor dynamics to embed interest rate trends into the dynamics of the yield curve, and assumes that the economy oscillates between the normal and ZLB regimes. The regimes are identified in our model via the first and second moments of the short rate; at the ZLB regime, the level of the short rate is near 0 to reflect a binding ZLB and its volatility is compressed. In order to ensure that the compression of short rate volatility to be transmitted through the short end of the yield curve, we let the short rate itself serve as one of the three latent factors driving the yield dynamics.

Empirically, the FS-ZLB model fits the yield curve quite well, generating measurement errors comparable to that of the FS-only model. Our model also demonstrates considerable forecasting promise, generating much higher posterior predictive densities out of sample compared to the Bauer and Rudebusch (2020) model with only falling stars and beating both the FS-only model and the random walk when it comes to forecasting the short rate. Despite allowing for regime-dependent dynamics, the FS-ZLB model retains an in-sample fit comparable to that of the FS-only model while generating more accurate out-of-sample short-rate forecasts, particularly during lower-bound periods.

1.1 Related Literature and Modeling Choices

Aside from the class of ATSMs with falling stars, our model represents the latest in an extensive collection of term structure models that seek to incorporate features of the ZLB. In contrast to a shadow-rate model, the FS-ZLB model does not enforce a hard floor on the short rate, instead allowing it to drift to a negative level albeit with very low

probability. The choice to sacrifice strict enforcement of the lower bound for a Markov-switching approach is motivated by two reasons, one technical and the other economic. Technically, shadow-rate models in the tradition of Wu and Xia (2016) employ a number of approximations to derive tractable expressions for bond yields, and require the use of the computationally intensive extended Kalman filter for estimation. The FS-ZLB model is instead only marginally more computationally costly compared to the FS-only model, and also admits exact closed-form bond yield formulas.

Furthermore, there is evidence that the shadow-rate specification is insufficient to capture the rich array of structural changes that accompany periods where the ZLB is binding. Christensen (2013) specifically points out that a regime-switching model better captures the compression of yields compared to a shadow-rate model, while Kim and Pribsch (2024) provides empirical evidence that shadow-rate models are structurally unstable, suggesting that there exists a shift in term structure dynamics during ZLB periods that are not fully captured by shadow-rate models.

Closely related to the FS-ZLB model are the models in Christensen (2013) and Hördahl and Tristani (2019), which also take a Markov-switching approach to the ZLB. While they utilize time-varying transition probabilities under which the probability of entering into the lower bound regime grows the closer the short rate is to the lower bound, the transition probabilities of the FS-ZLB model are constant. Again, this choice is motivated by technical and economic reasons. Over our sample, which ranges from 1985 to 2023, there are only four regime shifts that are observed with high probability, corresponding to the endpoints of the global financial crisis and COVID periods. The paucity of observed regime shifts indicate that the parameters related to the time-varying transition probabilities will not be well-identified, as pointed out in Hördahl and Tristani (2019).

A technical difficulty with time-varying transition probabilities is the loss of the ability to estimate the TP via a recursion that alleviates computational burdens. Below we show

that the loadings of the factors on the TP require the computation of expected values up to 120 periods ahead. Since this would normally require the summation of 210 components even when evaluating expectations for only 10 periods in the future, it is customary to turn to numerical methods. Fortunately, as we show in the appendix, the assumption of constant transition probabilities means that the regime process in our model is memoryless—and this allows us to collapse the 2120 computations involved to $2 \times 120 = 240$ computations, which greatly accelerates the sampling process and produces exact estimates of the TP given parameter estimates.

There is also an economic reason for allowing lower-bound entry risk to remain positive away from the lower bound. The recent spell of the ZLB induced by the pandemic has reminded the world that a binding interest rate lower bound is not a phenomenon confined to the Great Financial Crisis (GFC). The fact that the short rate plummeted from around 2% to a near-zero level in just over two months at the onset of the pandemic casts doubt as to whether the probability of entering into the lower bound regime is as closely tied to the level of the short rate as we previously suspected. There is an argument to be made about how, after being subjected to the GFC and the pandemic in quick succession, economic agents will always believe that there is some possibility that central banks to drastically cut rates in response to another economic crises, regardless of the current level of the short rate. Therefore, we treat entry into the lower-bound regime as carrying a positive probability that does not vanish when the current short rate is away from zero. This feature is also central to one of our main empirical findings: because lower bound entry risk remains positive in the normal regime, the possibility of future regime shifts lowers expected future short rates and raises the estimated TP even before the lower bound is reached.

The remainder of the paper is structured as follows. Section 2 introduces our model and the resulting expressions for bond yields and the TP. Section 3 discusses the estimation methodology and the simplifications we use to facilitate estimation. Section 4 presents our

empirical findings. Finally, Section 5 concludes.

2 A Regime-Shifting Model of Falling Stars and the Zero Lower Bound

Our model is based on Niu and Zeng (2012)'s discrete-time formulation of Christensen, Diebold, and Rudebusch (2011)'s AFNS model. We introduce in turn our specification for the risk-neutral dynamics, physical dynamics, and market prices of risk. The normal regime is denoted by $s_t = 1$, and the lower bound regime by $s_t = 2$.

2.1 The Risk-Neutral Dynamics

Zero-coupon bond yields are determined by three factors: the level factor L_t , the slope factor S_t , and the curvature factor C_t . Letting $f_t = (L_t, S_t, C_t)'$ be a vector collecting these factors, we first assume that the short rate r_t is an affine function of these factors:

$$r_t = \beta' f_t,$$

The risk-neutral measure $\mathbb{Q}$ is defined as the probability measure under which we have the risk-neutral no-arbitrage condition¹

$$P_t(\tau) = \mathbb{E}_t^{\mathbb{Q}}[\exp(-r_t)P_{t+1}(\tau - 1)], \quad (2.1)$$

where $P_t(\tau)$ is the (possibly regime-dependent) price at time t of a τ -maturity zero coupon bond with face value normalized to one, and $Y_t(\tau) := -\frac{1}{\tau} \log(P_t(\tau))$ is the yield of the same zero coupon bond. Under the $\mathbb{Q}$ -measure, the regime process s_t is Markovian with

¹The existence of $\mathbb{Q}$ under the no-arbitrage assumption is shown in, among others, Harrison and Kreps (1979).

time-invariant transition probability P^2 . If we have N regimes, for any $1 \leq i, j \leq N$ we define

$$P_{ij} = \mathbb{Q}(s_{t+1} = j \mid s_t = i).$$

Meanwhile, the factors f_t are assumed to follow a VAR(1) process

$$f_t = K_{s_t}^{\mathbb{Q}} + G^{\mathbb{Q}} f_{t-1} + \Omega_{s_t}^{\frac{1}{2}} \cdot v_t^{\mathbb{Q}}, \quad (2.2)$$

$$v_t^{\mathbb{Q}} \mid \mathcal{F}_{t-1}, s_t \sim \mathcal{N}(O_{3 \times 1}, I_3) \text{ under } \mathbb{Q}, \quad (2.3)$$

where $\mathcal{F}_t$ is the information available at time t .

Under the above specifications for the short rate and risk-neutral dynamics, bond prices are exponential-affine functions of the factors:

$$P_t(\tau) = \exp(-a_{s_t}(\tau) - b(\tau)' f_t), \quad (2.4)$$

$$a_{s_t}(\tau) = \mathbb{E}_t^{\mathbb{Q}} \left[a_{s_{t+1}}(\tau - 1) + b(\tau - 1)' K_{s_{t+1}}^{\mathbb{Q}} - \frac{1}{2} b(\tau - 1)' \Omega_{s_{t+1}} b(\tau - 1) \right] \quad (2.5)$$

$$b(\tau) = \beta + G^{\mathbb{Q}} b(\tau - 1), \quad (2.6)$$

where $a_{s_t}(\cdot)$ and $b(\cdot)$ are scalar and n -dimensional vector valued functions, respectively, with initial values given as $a_{s_t}(0) = 0$ and $b(0) = O_{n \times 1}$, respectively. These imply that the yields are determined as

$$Y_t(\tau) = \frac{a_{s_t}(\tau)}{\tau} + \frac{b(\tau)'}{\tau} f_t,$$

or as affine functions of the factors f_t . The recursive solutions for $a_{s_t}(\cdot)$ and $b(\cdot)$, as well as their closed-form versions, are obtained from the first-order approximation in Bansal and

²We assume, as is customary, that s_{t-1} is a sufficient statistic for s_t . In other words, given s_{t-1} , no other variable contains additional information about s_t . Throughout this paper, time t expectations are defined as expectations given all the information up to time t , including that of the factors and regimes; the dependence on s_t is emphasized when necessary.

Zhou (2002), and are presented in the appendix.

2.1.1 Modified AFNS Factor Loadings

It can be inferred from Equation (2.6) that the exact forms of the factor loadings $\frac{b(\tau)}{\tau}$ depend on the specification of $G^{\mathbb{Q}}$ and β . In a traditional AFNS model, these factor loadings assume the form

$$\frac{b(\tau)}{\tau} = \left(1 \quad \frac{1 - \exp(-\tau \kappa^{\mathbb{Q}})}{\tau \kappa^{\mathbb{Q}}} \quad \frac{1 - \exp(-\tau \kappa^{\mathbb{Q}})}{\tau \kappa^{\mathbb{Q}}} - \exp(-\tau \kappa^{\mathbb{Q}}) \right)', \quad (2.7)$$

where $\kappa^{\mathbb{Q}}$ is a decay parameter that determines the peak of the loading on the curvature factor (see, for example, Diebold and Li (2006))³. The parameters β and $G^{\mathbb{Q}}$ are then chosen such that the recursion (2.6) yields equation (2.7).

In Christensen, Diebold, and Rudebusch (2011), the short rate r_t , which in their study represents the instantaneous risk-free return, is given as

$$r_t = L_t + S_t,$$

or the sum of the level and the slope factors. Under this specification, identifying the lower bound regime based on the level and volatility of the short rate r_t requires us to impose restrictions on the sum $L_t + S_t$. This proves quite challenging in practical applications.

To sidestep this difficulty, we modify the factor loadings in the AFNS model so that

³Given the long sample period, one could allow the decay parameter to be time-varying, for instance by modeling it as a fourth (or fifth) factor as in Koopman, Mallee, and Van der Wel (2010) or Choi and Kang (2023). However, it is well-known that the sample does not contain much information about the decay parameter for it to be reliably estimated to begin with, making it difficult to accurately pin down the value of the decay parameter. Furthermore, in both methods mentioned above, the estimation of the time-varying decay parameter(s) require the use of the extended Kalman filter, which involves approximations and computational burdens that we wish to avoid.

A possible alternative to letting the decay parameter be fully time-varying is to let it exhibit regime-switching behavior. However, we find that there is insufficient evidence to conclude that the decay parameter switches between regimes in our model, so for the sake of parsimony we let it be time-invariant across the entire sample.

the level factor is identified with the short rate, or $L_t = r_t$. This means that for very short maturities, the loading on the level factor should be one, and the loadings on all other factors should be zero. One possible set of factor loadings that satisfies this criterion is given as

$$\frac{b(\tau)}{\tau} = \left(1 \quad 1 - \frac{1 - \exp(-\tau\kappa^{\mathbb{Q}})}{\tau\kappa^{\mathbb{Q}}} \quad \frac{1 - \exp(-\tau\kappa^{\mathbb{Q}})}{\tau\kappa^{\mathbb{Q}}} - \exp(-\tau\kappa^{\mathbb{Q}}) \right)', \quad (2.8)$$

and the values of β and $G^{\mathbb{Q}}$ the result in the above form are given by⁴

$$\beta = \left(1 \quad 1 - \frac{1 - \exp(-\kappa^{\mathbb{Q}})}{\kappa^{\mathbb{Q}}} \quad \frac{1 - \exp(-\kappa^{\mathbb{Q}})}{\kappa^{\mathbb{Q}}} - \exp(-\kappa^{\mathbb{Q}}) \right)' \quad (2.9)$$

$$G^{\mathbb{Q}} = \begin{pmatrix} 1 & 1 - \exp(-\kappa^{\mathbb{Q}}) & \kappa^{\mathbb{Q}} \exp(-\kappa^{\mathbb{Q}}) \\ 0 & \exp(-\kappa^{\mathbb{Q}}) & -\kappa^{\mathbb{Q}} \exp(-\kappa^{\mathbb{Q}}) \\ 0 & 0 & \exp(-\kappa^{\mathbb{Q}}) \end{pmatrix}. \quad (2.10)$$

β and $G^{\mathbb{Q}}$ are specified as in equations (2.9) and (2.10) going forward, and assume regime-shifting dynamics only for the level factor, or the short rate. In particular, we let the innovations to the level factor and the other two factors be independent:

$$\Omega_{s_t}^{\frac{1}{2}} = \begin{pmatrix} \underbrace{\omega_{s_t}}_{1 \times 1} & O_{1 \times 2} \\ O_{2 \times 1} & \underbrace{\Omega_{22}^{\frac{1}{2}}}_{2 \times 2} \end{pmatrix}.$$

The assumption of independent innovations does not necessarily mean that the level factor itself is independent of the other two factors. For example, they may be correlated if $G^{\mathbb{Q}}$ is

⁴In Bauer and Rudebusch (2020), it is mentioned that the risk-neutral factor dynamics are kept stationary to avoid the counterfactual implication of yields that diverge to $-\infty$ as the maturity goes to infinity. This is due to the Jensen's inequality term in the intercept $\frac{a_{s_t}(\tau)}{\tau}$ diverging to $-\infty$.

In our model, the unit root in $G^{\mathbb{Q}}$ renders the risk-neutral factor dynamics non-stationary, so that yields also diverge to $-\infty$ as the maturity τ goes to infinity. This is an undeniable shortcoming of our model. However, the speed at which the Jensen's inequality term in the intercept above diverges is not too fast, with $\frac{a_{s_t}(\tau)}{\tau}$ on average increasing in τ at least until $\tau = 180$, which is the highest maturity in our sample. Therefore, we retain the unit root under the risk-neutral measure despite its counterfactual implications, in light of the many advantages conferred by the Nelson-Siegel specification.

not diagonal, as in our AFNS model. Furthermore, this assumption is necessary to identify the model, as shown in the appendix.

2.2 The Physical Dynamics

The physical dynamics of the model establish a connection between the yields and risk factors. As stated earlier, the regime-dependent parameters are linked to the physical dynamics of the level factor. These dynamics depart from the typical factor dynamics specification in two significant ways: they incorporate the cointegrating relationship among yields, as well as structural changes that arise when the lower bound is binding.

The structural changes that arise from a binding lower bound are captured by switching between the normal and lower bound regimes. These regimes are in turn identified using the first two moments of the level factor, which in this case is simply the short rate r_t . Specifically, we identify as belonging to the lower bound regime periods where the short rate is near 0 and its volatility is very small. The second identifying feature embeds the compression of yield volatility into the lower bound regime, where the low volatility of the level factor is transmitted to yields of higher maturities through the bond pricing equation.

The choice to identify the lower bound regime via the first two moments of the short rate also imply important changes in the persistence of the yield factors across the two regimes. Two well documented facts in the literature are that the real interest rate contains a trending component (e.g. Del Negro et al. (2019)), through which the nominal interest rate, or the short rate, is also rendered non-stationary; and that a collection of T yields are cointegrated of order $T - 1$ (a classical treatment can be found in Hall, Anderson, and Granger (1992)). Taken together, these observations imply that the short rate contains the single common stochastic trend shared by yields of all maturities⁵. Since the short rate itself

⁵We emphasize that this does not mean that we are equating the short rate with the its trend component i^* . i^* is not the immediate object of interest in this paper, so we do not separately identify the trend and cyclical components of the short rate.

enters into our model as a yield factor, this means that our model contains two stationary factors and one non-stationary factor in the form of the short rate. However, during the ZLB regime, for the first and second moments of the short rate to be well-defined, the short rate must itself be stationary. Therefore, the yield factors are $I(1)$ during the normal regime with the short rate serving as the single common trend, whereas they are $I(0)$ during the ZLB regime.

In our model, the $\mathbb{P}$ -dynamics of the common factors f_t are governed by the VAR system⁶

$$f_t = \underbrace{\begin{pmatrix} \rho_{st} & G_{12,s_t}^{\mathbb{P}} & G_{13,s_t}^{\mathbb{P}} \\ 0 & G_{22}^{\mathbb{P}} & G_{23}^{\mathbb{P}} \\ 0 & G_{32}^{\mathbb{P}} & G_{33}^{\mathbb{P}} \end{pmatrix}}_{G_{s_t}^{\mathbb{P}}} f_{t-1} + \underbrace{\begin{pmatrix} \omega_{st} & O_{1 \times 2} \\ O_{2 \times 1} & \Omega_{22}^{\frac{1}{2}} \end{pmatrix}}_{\Omega_{s_t}^{\frac{1}{2}}} v_t^{\mathbb{P}}, \quad (2.11)$$

$$v_t^{\mathbb{P}} \mid \mathcal{F}_{t-1}, s_t \sim \mathcal{N}(O_{3 \times 1}, I_3) \quad \text{under the physical measure,}$$

$$\max \left| \text{eig} \begin{pmatrix} G_{22}^{\mathbb{P}} & G_{23}^{\mathbb{P}} \\ G_{32}^{\mathbb{P}} & G_{33}^{\mathbb{P}} \end{pmatrix} \right| < 1,$$

$$\omega_1^2 > \omega_2^2,$$

$$\rho_1 = 1, \quad |\rho_2| < 1,$$

$$G_{12,2}^{\mathbb{P}} = G_{13,2}^{\mathbb{P}} = 0,$$

$$s_t \sim \text{Markov}(s_0, P).$$

Equation (2.11) gives a maximally flexible specification for the $\mathbb{P}$ -dynamics that preserves two empirical features: that the yield factors are jointly $I(1)$ with exactly two cointegrating relationships (one common trend) among them, and that the short rate exhibits minimal

⁶The last condition lets the regime process s_t be a Markov process with the same (constant) transition probability P under $\mathbb{P}$ as under $\mathbb{Q}$. We also assume that s_{t-1} remains a sufficient statistic for s_t under the physical measure.

volatility around 0 during the lower bound regime. In the appendix it is shown that the block upper triangular form of $G_1^{\mathbb{P}}$ and the eigenvalue constraints presented above are necessary and sufficient for the factors f_t to be jointly I(1) with exactly 2 cointegrating relationships in the normal regime. For the level factor to be stationary in the lower bound regime, it must be the case that $|\rho_2| < 1$. $\omega_2^2 < \omega_1^2$ is the mathematical formulation for the compression of yield volatility during the ZLB regime. Finally, $G_{12,2}^{\mathbb{P}} = G_{13,2}^{\mathbb{P}} = 0$ is imposed to prevent covariation with the slope and curvature factors from inflating the variance of the level factor during the lower bound regime.

2.3 The Term Premium

In this section, we connect the $\mathbb{Q}$ -and $\mathbb{P}$ -dynamics introduced earlier in the form of the TP of a τ -maturity bond.

The term premium $TP_t(\tau)$ for a τ -maturity bond at time t is the difference between the τ -maturity yield and the expectations hypothesis (EH). Formally, because the τ -maturity yield is equal to the average expected path of the short rate when investors are risk neutral, the τ -maturity TP at time t is defined as

$$TP_t(\tau) = Y_t(\tau) - \frac{1}{\tau} \sum_{i=0}^{\tau-1} \mathbb{E}_t[r_{t+i}], \quad (2.12)$$

where the second term represents the EH part of $Y_t(\tau)$. Cochrane and Piazzesi (2008) show that the TP at time t for a τ -maturity bond can be expressed as

$$TP_t(\tau) = \frac{1}{\tau} \sum_{i=0}^{\tau-2} \mathbb{E}_t \left[\text{err}_{t+i+1}^{(\tau-i)} \right], \quad (2.13)$$

or the average expected one-period excess return throughout the life of the bond. Therefore, using the formula above with the law of iterated expectations, the TP can be expressed as

the sum of two components: the constant part

$$\frac{1}{\tau} \sum_{i=0}^{\tau-2} \left(b(\tau-i-1)' \mathbb{E}_t \left[K_{s_{t+i+1}}^{\mathbb{Q}} \right] - \frac{1}{2} b(\tau-i-1)' \mathbb{E}_t \left[\Omega_{s_{t+i+1}} \right] b(\tau-i-1) \right) \quad (2.14)$$

and the factor part

$$\frac{1}{\tau} \sum_{i=0}^{\tau-2} \left(-b(\tau-i-1)' \mathbb{E}_t \left[\left(G_{s_{t+i+1}}^{\mathbb{P}} - G^{\mathbb{Q}} \right) f_{t+i} \right] \right). \quad (2.15)$$

Given that these terms depend on s_t , we are interested in the TP associated with the current regime, that is, the compensation investors require for holding a bond with a long maturity when they are aware of the current state of the economy.

3 Estimation

We use Bayesian Markov Chain Monte Carlo (MCMC) methods to estimate the regime process and parameters due to the irregularity of the likelihood. Specifically, we assume informative priors for some parameters that are difficult to estimate because of insufficient information in the data, which helps to regularize the posterior surface around the posterior mode. Our sample includes the short rate and U.S. yields for $m = 17$ maturities: 3 and 6 months, and yearly maturities from 1 to 15 years. Letting $\tau_1, \dots, \tau_m$ be the maturities in our sample apart from the short rate, the sample of time t yields of maturity 3 months or higher are collected in the vector

$$\mathcal{Y}_t = \begin{pmatrix} Y_t(\tau_1) \\ \vdots \\ Y_t(\tau_m) \end{pmatrix}.$$

The model-implied yield formula is

$$\mathcal{Y}_t = \mathcal{A}_{s_t} + \mathcal{B}f_t, \quad \mathcal{A}_{s_t} = \begin{pmatrix} \frac{a_{s_t}(\tau_1)}{\tau_1} \\ \vdots \\ \frac{a_{s_t}(\tau_m)}{\tau_m} \end{pmatrix} \quad \text{and} \quad \mathcal{B} = \begin{pmatrix} \frac{b(\tau_1)'}{\tau_1} \\ \vdots \\ \frac{b(\tau_m)'}{\tau_m} \end{pmatrix} \in \mathbb{R}^{m \times 3}.$$

To complete the model, we assume the presence of a short rate measurement error u_t with a variance of σ_u^2 , and a vector of measurement errors e_t pertaining to the other sample yields. As is standard in the literature, we assume the measurement errors for the yields are mutually independent and have variances $\sigma_{e,1}^2, \dots, \sigma_{e,m}^2$. Our model is thus comprised of the measurement equations

$$r_t = \beta' f_t + \sigma_u \cdot u_t$$

$$\mathcal{Y}_t = \mathcal{A}_{s_t} + \mathcal{B}f_t + \Sigma_e^{\frac{1}{2}} \cdot e_t,$$

where $\Sigma_e = \text{diag}(\sigma_{e,1}^2, \dots, \sigma_{e,m}^2)$, and the transition equation

$$f_t = \underbrace{\begin{pmatrix} \rho_{s_t} & g_{s_t}^{\mathbb{P}'} \\ O_{2 \times 1} & \tilde{G} \end{pmatrix}}_{G_{s_t}^{\mathbb{P}}} f_{t-1} + \Omega_{s_t}^{\frac{1}{2}} \cdot v_t^{\mathbb{P}}.$$

In the estimation process, all yields are appropriately scaled via the scaling parameter $c = 1200$ to transform them into per annum percentage values.

We impose the following restrictions on this baseline state-space model. First, we put $\beta = (1, 0, 0)'$, so that we essentially identify the level factor with the short rate. Next, we follow the lead of Hamilton and Wu (2012) and the RY specification of case **P** in Joslin, Singleton, and Zhu (2011), and assume that the short rate, the 3 year yield and

the 10 year yield are observed without error. This allows us to recover the level, slope and curvature factors as a linear combination of the three yields that are observed without error. Subsequently, we are able to avoid the complications that arise from running a Kalman filtering and smoothing process to recover the latent factors.

More importantly, it is pointed out in Hamilton and Wu (2012) that these simplifications allow us to represent the state space model above as a restricted Gaussian VAR system. In our case, this system is characterized by the following three equations:

$$r_t = -g_{s_t}' \mathcal{B}^{(1)-1} \mathcal{A}_{s_{t-1}}^{(1)} + \rho_{s_t} r_{t-1} + g_{s_t}' \mathcal{B}^{(1)-1} \cdot \mathcal{Y}\mathcal{S}_{t-1}^{(1)} + \omega_{s_t} \cdot v_{1t}^{\mathbb{P}} \quad (3.1)$$

$$\mathcal{Y}\mathcal{S}_t^{(1)} = \left(\mathcal{A}_{s_t}^{(1)} - \mathcal{B}^{(1)} \tilde{G} \mathcal{B}^{(1)-1} \cdot \mathcal{A}_{s_{t-1}}^{(1)} \right) + \mathcal{B}^{(1)} \tilde{G} \mathcal{B}^{(1)-1} \cdot \mathcal{Y}\mathcal{S}_{t-1}^{(1)} + \mathcal{B}^{(1)} \Omega_{22}^{\frac{1}{2}} \cdot v_{SC,t}^{\mathbb{P}} \quad (3.2)$$

$$\mathcal{Y}\mathcal{S}_t^{(2)} = \left(\mathcal{A}_{s_t}^{(2)} - \mathcal{B}^{(2)} \mathcal{B}^{(1)-1} \mathcal{A}_{s_t}^{(1)} \right) + \mathcal{B}^{(2)} \mathcal{B}^{(1)-1} \cdot \mathcal{Y}\mathcal{S}_t^{(1)} + \Sigma_{\tilde{e}}^{\frac{1}{2}} \cdot e_t, \quad (3.3)$$

where $\mathcal{Y}\mathcal{S}_t^{(1)}$ collects the yield spreads of the 3 year and 10 year yield, and $\mathcal{Y}\mathcal{S}_t^{(2)}$ the yield spreads of the remaining sample yields. The first two equations represent the dynamic part of the system and the third the static part. The three equations above can be taken together to form a larger reduced-form VAR system

$$X_t = C_{1,s_t,s_{t-1}} \cdot \mathcal{A} + \Phi_{s_t} \cdot X_{t-1} + \Sigma_{s_t} \cdot u_t, \quad (3.4)$$

$$X_t = \left(r_t, \mathcal{Y}\mathcal{S}_t^{(1)}, \mathcal{Y}\mathcal{S}_t^{(2)} \right)$$

$$\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2).$$

The parameters $C_{1,s_t,s_{t-1}}$, Φ_{s_t} and Σ_{s_t} are defined in the appendix. We now employ a two-step procedure to obtain a sample from the posterior distribution of the structural

parameters

$$\theta = \{\rho_2, g_1^{\mathbb{P}}, \omega_1^2, \omega_2^2, \tilde{G}, \Omega_{22}, \kappa^{\mathbb{Q}}, K_1^{\mathbb{Q}}, K_2^{\mathbb{Q}}, \Sigma_e, P\}.$$

First, we obtain a sample from the posterior distribution of the reduced form parameters

$$\theta^* = \{\rho_2, g_1^{\mathbb{P}}, \omega_1^2, \omega_2^2, \tilde{G}, \Omega_{22}, \kappa^{\mathbb{Q}}, \mathcal{A}_1, \mathcal{A}_2, \Sigma_e, P\}$$

that are present in the reduced form model above via a simple Gibbs sampling approach. Then, we recover the structural parameters by transforming the sample of reduced form parameters. This can be seen as a version of minimum chi-square estimation, and the frequentist validity of this procedure implies Bayesian validity through the Bernstein-von Mises (BvM) theorem.

4 Empirical Results

We now present our empirical results. First, we show how well the model fits the yield curve, particularly during periods when the ZLB is binding. We then discuss how accounting for both falling stars and the ZLB affects estimates of the term premium.

4.1 Model Evaluation

In this section, we evaluate our model across various dimensions, from the size of the fitted errors to forecast performance. In particular, we compare the performance of our FS-ZLB model, which includes both falling stars and the ZLB, against the FS-only model, which includes only falling stars⁷. We run 10,000 iterations of the Gibbs sampler with a burn-in

⁷The FS-only model is a special case of the FS-ZLB model with transition probability equal to the identity matrix, and we estimate it as such.

Table 1: **Fitted Errors** Tabulated below are the RMSEs in basis points of the fitted yields across chosen maturities.

	Maturities in Months									Average
	3	6	12	24	36	60	84	120	180	
FS-ZLB	42.3	37.58	19.92	9.86	4.81	6.29	5.81	1.56	10.84	10.53
FS-Only	40.7	35.16	18.23	10.32	5.45	5.71	5.89	3.31	10.52	10.58

of 1,000 iterations to estimate the parameters of the FS-ZLB model and FS-only model.

Below the models are compared across two dimensions: the ability to fit the observed yields and short rate forecasting ability. In the appendix, we offer additional indicators of satisfactory performance of the FS-ZLB model, including its ability to accurately capture the yield compression phenomenon.

4.1.1 Yield Fit

We obtain a sample of the model-implied τ maturity yield by computing

$$\frac{a_{st}(\tau)}{\tau} + \frac{b(\tau)'}{\tau} f_t,$$

using the values of the parameters procured in each iteration of the Gibbs sampling procedure.

Table 1 presents the FS-ZLB model's root mean-squared error (RMSE) across all maturities, compared to the corresponding RMSEs of the FS-only model. The RMSEs are defined here as

$$\sqrt{\frac{1}{T} \sum_{t=1}^T \left(Y_t(\tau) - \frac{a_{st}(\tau)}{\tau} - \frac{b(\tau)'}{\tau} f_t \right)^2}.$$

It is immediately noticeable that the two models both fit the yield curve extremely well, incurring a mean RMSE of around 10 basis points. There are very little differences in yield

Table 2: **RMSE from Short Rate Forecasts** Tabulated below are the RMSEs in basis points of the out-of-sample short rate forecasts obtained from each model over the 20 year period from January 2002 to December 2021. Also reported are the RMSEs over the heuristic normal regime (short rate greater than or equal to 25 basis points) and the heuristic lower bound regime (short rate lower than 25 basis points).

	Forecast Horizon		
	3 Months	6 Months	1 Year
<i>Full Sample</i>			
Random Walk	37.49	65.73	126.33
FS-Only	32.98	57.87	107.26
FS-ZLB	28.05	51.22	101.26
<i>Normal Regime: $r_t \geq 0.25$</i>			
Random Walk	37.36	65	116.82
FS-Only	27.61	50.06	89.67
FS-ZLB	27.68	50.29	89.49
<i>Lower Bound Regime: $r_t < 0.25$</i>			
Random Walk	3.14	9.79	48.09
FS-Only	18.04	29.03	58.86
FS-ZLB	4.56	9.73	47.37

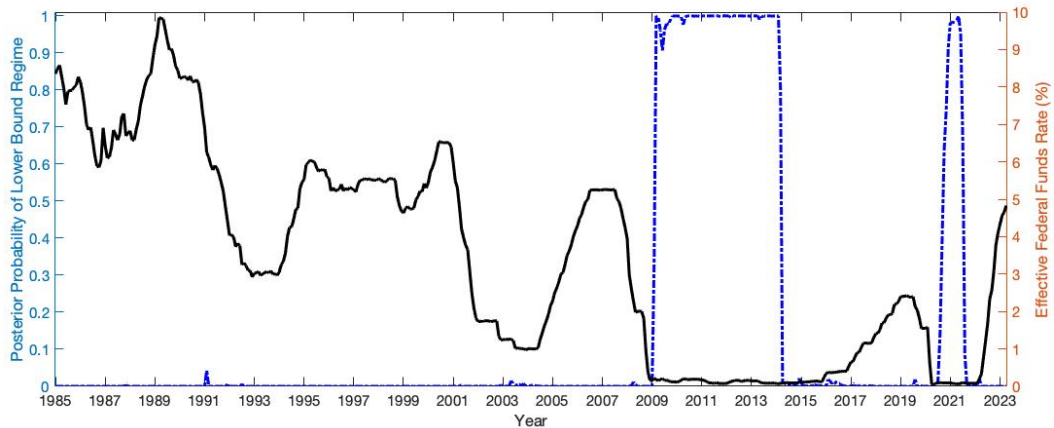
fit for the two models.

Notably, the RMSEs for yields of maturities 3 years and 10 years, though the smallest among the maturities presented in Table 1, are nonetheless non-zero. At first glance, this seems to contradict our assumption that the 3 and 10 year yields are measured without error. This does not represent a contradiction in our model, but instead reflects the fact that, due to our two-step estimation approach, the estimator of $\mathcal{A}_{s_t}$ only asymptotically satisfies the no-arbitrage restrictions in Equation (2.5).

4.1.2 Estimated Regimes

Figure 1 displays the posterior probability of the economy being in the lower bound regime. Our model reports that investors believe that the GFC periods from January 2009 to mid 2014, along with the periods from early 2020 to early 2022 that correspond to the COVID-

Figure 1: **Posterior Probability of the ZLB Regime** This figure displays the posterior probabilities that the economy is at the lower bound regime. The dashed line represents the posterior probabilities, and the solid line is the federal funds rate.

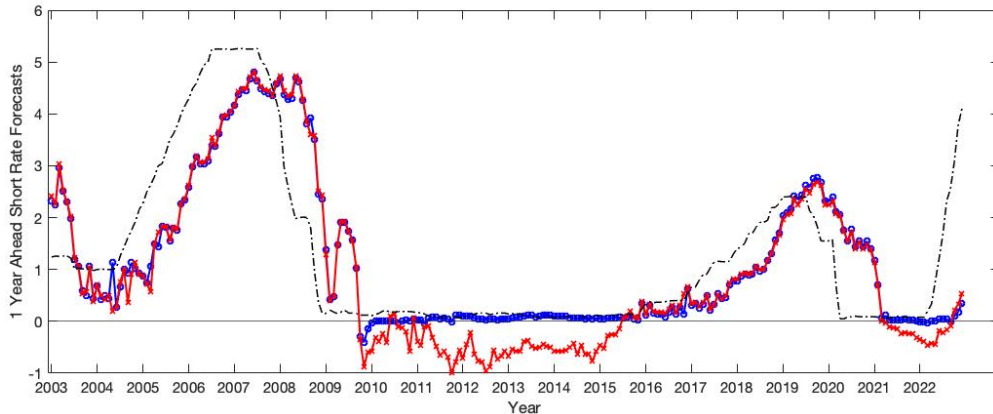


19 pandemic, belong to the lower bound regime with a high probability. It is largely agreed upon that the lower bound was binding during these periods, demonstrating our model's ability to accurately capture the ZLB periods. Notably, the small drop in the posterior probability around mid to late 2013 may reflect the taper tantrum.

In contrast to Hördahl and Tristani (2019), who report that the U.S. economy belongs to the lower bound regime until as late as November 2015, in our model the economy lifts off from the ZLB around the middle of 2014. This discrepancy reflects the differences in how the lower bound regime is identified in our model compared to Hördahl and Tristani (2019). In the latter, the time-varying transition probability of being in the lower bound regime depends on the level of the short rate, so that any period in which the short rate is near 0 is identified as belonging to the lower bound regime with a high probability. In fact, Hördahl and Tristani (2019) point out that their the liftoff date of November 2015 corresponds to when the federal funds rate first started to increase since 2009.

Meanwhile, in our model the level of yields of longer maturities, alongside the level of the short rate, are taken into consideration through the parameter $\mathcal{A}_{st}$ when obtaining the posterior probabilities of being in the lower bound regime. While the short rate would not

Figure 2: **One-Year-Ahead Short Rate Forecasts** This figure plots the one-year-ahead out-of-sample forecast of the short rate. The round markers represent the forecasts of the FS-ZLB model, and the crossed markers those of the FS-only model. The dotted-dashed line is the effective federal funds rate.



lift off from near 0 until late 2015, yields of maturities 2 to 10 years start to rise around late 2014, which is precisely when the Fed halted the QE program. As such, our model reports that the economy lifts off from the lower bound as soon as the QE program is halted.

4.1.3 Short Rate Forecasts

The term premium of a long term yield is the difference between the yield and its expectation hypothesis component, where the latter is again the average expected short rate over the life of the bond. Therefore, the more accurate the forecasts of the short rate, the more likely it is to obtain reliable term premium estimates. To compare the forecasting abilities of the FS-only and FS-ZLB models, we perform an out-of-sample forecasting exercise in which we use each model to forecast the short rate each month over the 20 year period from January 2002 to December 2021. We produce these forecasts for horizons of three, six and twelve months. Also compared against the FS-only and FS-ZLB models is the simple random walk specification, where the h -period ahead short rate r_{t+h} is forecast as r_t at time t .

The measure of forecasting ability is the RMSE, where the RMSE for the h -period

forecast is defined as

$$\sqrt{\frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1} (r_t - \mathbb{E}_t[r_{t+h}])^2},$$

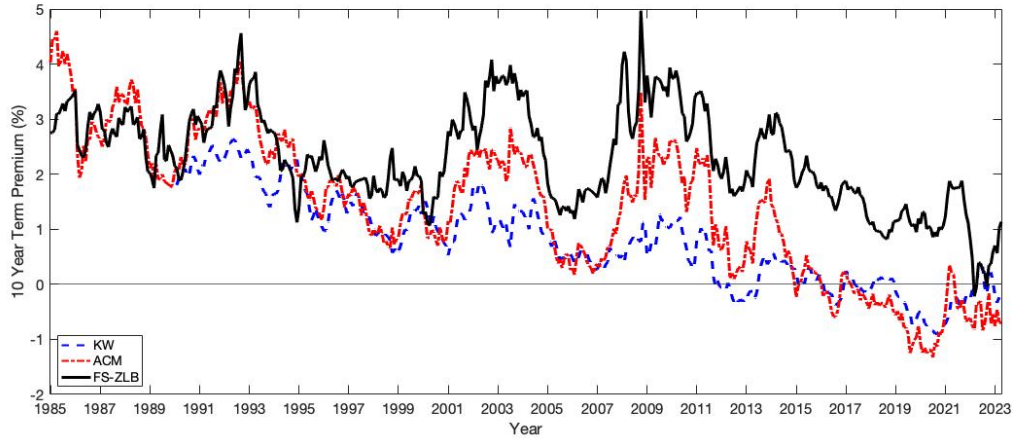
letting t_0 be the start of the out-of-sample period and t_1 its end. At time t , the forecast $\mathbb{E}_t[r_{t+h}]$ is computed for each model according to the method outlined in Appendix H.

Table 2 presents the RMSEs from the out-of-sample forecasts for the FS-only and FS-ZLB models, over both the full sample and subsamples corresponding to the normal and lower bound regimes. These subsamples are heuristically determined according to whether the short rate exceeds 25 basis points. In the full sample, the FS-only and FS-ZLB models beat the random walk specification, and the FS-ZLB model consistently dominates the FS-only model, across all forecast horizons. This demonstrates that the FS-ZLB model significantly improves upon the FS-only model when it comes to forecasting the short rate.

In the normal subsample, the FS-only and FS-ZLB models perform similarly and continue to beat the random walk model. In contrast, in the lower bound subsample, the performance of the FS-only model lags behind that of the random walk and FS-ZLB alternatives. The FS-ZLB model displays RMSEs comparable to the random walk specification in this subsample, but notably it still outperforms the random walk under horizons of six months and one year. The difference in forecast ability over the two subsamples suggests that the adherence of the FS-ZLB model to the ZLB restriction drives its superior forecasts.

Figure 2, which plots the one-year-ahead short rate forecasts obtained from each model, throws this difference into sharper relief. While the forecasts produced by the two models coincide for most of the sample, they diverge noticeably when the short rate is close to 0. In particular, the forecasts of the FS-ZLB model rarely become negative, whereas the FS-only model predicts negative short rates for years at a time. This shows how introducing regime-switching dynamics during lower bound periods in the FS-ZLB model improves its

Figure 3: **Term Premium Estimates** This figure compares the ACM, KW, and FS-ZLB estimates of the term premium of a zero coupon bond with maturity 10 years.



forecast ability compared to the FS-only model.⁸

Here, we showed how incorporating the ZLB into the FS-only model via regime switching improves short rate forecasts by imposing the restriction that the short rate cannot become highly negative. In the appendix, we also compare how the FS-ZLB model fares against the FS-only model from the point of view of overall forecasting ability, measured in terms of the posterior predictive likelihood (PPL). Results indicate that adherence to the ZLB restriction not only improves forecasts of the short rate, but also benefits forecasting ability in general.

4.2 Estimation of the Term Premium

In this section, we present findings related to the TP. We begin by comparing the TP estimate from our model with those derived from Gaussian ATSMs. Next, we concentrate on how the inclusion of the ZLB in addition to falling stars affects TP estimation.

⁸Kang (2015) reports similar results of out-of-sample density forecasting in the framework of the dynamic Nelson-Siegel model.

4.2.1 Comparison with Gaussian ATSMs

In Figure 3 we compare the term premia for the 10-year yield obtained from our model with two other widely used TP, namely, those of Kim and Wright (2005) (henceforth, KW)⁹ and Adrian, Crump, and Moench (2013) (henceforth, ACM).¹⁰ The TP formula in equations (2.14) and (2.15) involves complex expectations of future values at time t , which are challenging to evaluate using numerical methods. In the appendix, we present a simple algorithm that enables the analytical calculation of the quantities in (2.14) and (2.15). We calculate our term premium by employing this algorithm.

The FS-ZLB term premium estimate is clearly more stationary relative to the KW and ACM estimates, which exhibit a slow downward trend that ultimately reaches negative values toward the end of our sample period. This demonstrates that, as noted in Bauer and Rudebusch (2020), including the interest rate trend makes the TP more stationary and the expectations component less so¹¹. Notably, the level of the TP in our model consistently remains higher than the KW and ACM term premia and remains positive throughout the sample period. In this regard, the TP estimated in our model is similar to the estimates

⁹Provided by the FRED database; end-of-month averaged

¹⁰Provided by the New York FRB, available in monthly frequency

¹¹This does not mean, however, that our TP estimate is completely stationary. To understand why this is, note that the loading of the (non-stationary) level factor on the TP depends on the expectation of first column of the difference $G_{s_t}^{\mathbb{P}} - G^{\mathbb{Q}}$. This expectation is not always equal to 0 because the (1,1) element of $G_{s_t}^{\mathbb{P}}$ regime-switches between 1 and ρ_2 , where the latter is less than 1 in absolute value, whereas the (1,1) element of $G^{\mathbb{Q}}$ is always fixed at 1, thus inducing a non-zero loading of the level factor on the TP.

The result that the TP contains a degree of non-stationarity is in line with the findings in Bauer and Rudebusch (2020), where it is shown that, because the interest rate trend loads on the yields with a coefficient greater than unity, both the TP and expectations component are non-stationary. AFNS models with falling stars often assume that the common trend loads on the level factor with unity and that the slope and curvature factors are stationary, which means that the term premium estimates produced by these models are completely stationary and thus do not contain the slight downward trend that the term premium of Bauer and Rudebusch (2020) does. Although the common trend also loads on the level factor with unity in our model, we allow factor dynamics to switch and become stationary during the lower bound regime. Therefore our TP estimate ends up becoming slightly non-stationary. In light of the findings in Bauer and Rudebusch (2020), we feel confident in saying that this better reflects stylized facts about the yield curve.

Nevertheless, compared to the term premium estimates from Gaussian ATSMs, the TP obtained from our FS-ZLB model exhibits a considerably more subdued downward trend. This is also in line with the results in Bauer and Rudebusch (2020).

in both the B-CR model of Christensen and Rudebusch (2016) and the regime-switching model of Hördahl and Tristani (2019).

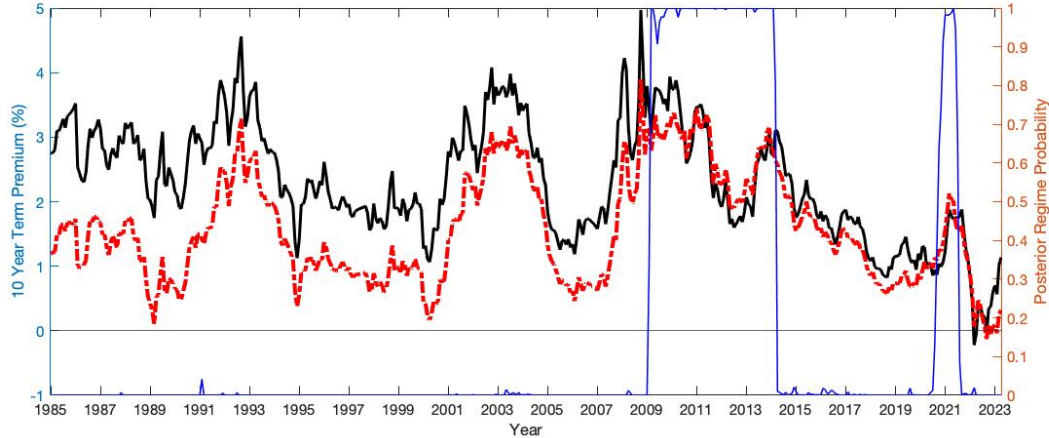
A more detailed comparison of the estimated term premia provides a clearer understanding of the differences between the FS-ZLB model and Gaussian ATSMs. In both models, the TP rises after the start of the GFC, followed by a sharp decrease in the early 2010s. This trend is likely a result of unconventional monetary policies, such as the Federal Reserve’s initial LSAPs (often referred to as QE1), which reduced long-term yields. All three models also show a modest rise in the TP in the mid-2010s, reflecting the effects of the taper tantrum of 2013.

Despite their similarities, the extent of the changes in the TP varies across models. In the KW and ACM models, the TP drops to nearly zero (and for the KW term premium, even below zero) during the early years of the GFC. In contrast, the TP calculated using the FS-ZLB model shows a less significant decrease, indicating that expectations for the short rate did not react as sharply to QE1 as indicated by the other models.

The gap between the KW and ACM term premia and the FS-ZLB term premium grows even larger after following the end of the GFC. While the expected short rate quickly returns to its historical positive level under Gaussian ATSM, the movement of short rate expectations is more tempered under the FS-ZLB model. In fact, the TP only falls below zero following the COVID-19 period, in contrast to the other models, which report negative term premia as early as 2015.

This indicates that the marked increase in the TP observed in our model, as opposed to traditional Gaussian ATSMs, can be attributed to the inclusion of falling stars. According to Ichiue, Ueno, et al. (2013) and Bauer, Rudebusch, and Wu (2012), in Gaussian ATSMs with stationary factors, the factors—and consequently the yields—tend to revert quickly to their constant long-term means. Since treasury yields are always positive, their means are significantly positive, leading to a gradual increase in the expected short rate to this

Figure 4: **Term Premium Estimates from Falling Stars Models** This figure compares the term premium estimates of a zero coupon bond with maturity 10 years obtained from the FS-ZLB and the FS-only models. The solid line represents the term premium estimate of the FS-ZLB model, and the dashed line that of the FS-only model. The thin solid line, meanwhile, is the estimated posterior regime probability.



positive value after the ZLB. In contrast, in an ATSM with falling stars, the physical factor dynamics are non-stationary, meaning that the short rate does not have to rise rapidly post-ZLB. In essence, Gaussian ATSMs underestimate the TP compared with models with falling stars when considering the slow adjustment of expectations.

4.2.2 Comparison with the FS-Only Model

The KW and ACM term premia are derived from ATSMs with stationary factor dynamics and constant factor volatility. This means that any difference between our model's TP estimate and the KW and ACM estimates could be imputed to the inclusion of either falling stars or changing dynamics at the ZLB. To isolate the effect of ZLB-induced structural changes on the TP, we compare and contrast the TP from our model against that from a model with only falling stars.

Figure 4 presents a comparison of the TP between the FS-ZLB and FS-only models. It indicates that consideration of structural changes entailed by a binding ZLB affects TP estimation not only during periods where the short rate is stuck at the lower bound, but

throughout the entire sample period. During the lower bound regime, which includes the period from early 2009 to 2014, the TP produced by the FS-ZLB model exhibits greater volatility compared to the TP of the FS-only model, with differences in both directions reaching up to 100 basis points. Notably, the FS-only model does not account for the temporary, yet significant increases in the TP observed from 2009 to 2010. Furthermore, the FS-only model overestimates the TP in the period from 2011 to 2013, right after QE1. Since long-term yields are decomposed into the EH and TP components, the underestimation of TP movements by the FS-only model implies an overestimation of the movements of short rate expectations during the ZLB period. This could potentially mislead economic actors, including central bankers and investors, by placing undue emphasis on the signaling effect of unconventional monetary policy over the portfolio balance effect.¹²

The differences in TP estimates between the two models are also observed during the normal regime. Figure 4 also suggests that the possibility of entering the lower bound regime also affects how term premia react before the economy actually reaches the ZLB. Throughout the periods that are identified as belonging to the normal regime, the TP produced by the FS-ZLB model is consistently higher than that reported by the FS-only model, while there seems to be no noticeable differences in volatility. In other words, agents in the FS-ZLB model expect future short rates to be lower than their counterparts in the FS-only model. This difference arises because, even during the normal regime, investors expect the economy to enter into the lower bound regime with a probability that increases the larger the forecast horizon.

4.2.3 Drivers of the Term Premium under the ZLB

So far, we have seen that the level of the TP differs between the FS-ZLB and FS-only models during the normal regime, while the volatility differs during the lower bound regime. As

¹²The signaling and portfolio balance channels of unconventional monetary policy are examined in Bauer and Rudebusch (2014).

Table 3: **Term Premium Factor Loadings** Tabulated below are the loadings of the slope and curvature factors on the term premium in the FS-ZLB and FS-only models for selected maturities. Standard errors are in parentheses.

Maturities	FS-ZLB Model				FS-only Model	
	Normal Regime		Lower Bound Regime		Slope	Curvature
	Slope	Curvature	Slope	Curvature		
24	0.032 (0.0943)	-0.164*** (0.0644)	0.3851*** (0.0247)	0.2516*** (0.0217)	0.0844 (0.0727)	-0.0611 (0.0501)
36	0.0625 (0.1362)	-0.195*** (0.0929)	0.4844*** (0.0399)	0.2603*** (0.03)	0.1188 (0.1055)	-0.0841 (0.0721)
60	0.132 (0.1829)	-0.2325** (0.1327)	0.6079*** (0.0596)	0.2294*** (0.0423)	0.171 (0.1471)	-0.1294 (0.1053)
84	0.1969 (0.2004)	-0.2578** (0.1553)	0.6817*** (0.0676)	0.1888** (0.052)	0.2062 (0.168)	-0.1692 (0.1281)
120	0.2744 (0.209)	-0.2798** (0.1723)	0.7481*** (0.071)	0.1403* (0.0612)	0.2382 (0.1849)	-0.2134* (0.1511)
180	0.367 (0.2123)	-0.287*** (0.1844)	0.8073*** (0.0713)	0.0931 (0.0697)	0.2645 (0.2013)	-0.2556* (0.1759)

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

explained above, the differences in the reported term premia during the normal regime can be chalked up to differences in how agents form expectations about the short rate. On the other hand, we argue that the volatility discrepancies observed during the lower bound regime can be attributed to differences in how agents perceive risk across the two models.

In the appendix, we show how the change in the parameter $g_{st}^{\mathbb{P}}$, which governs the loading of the slope and curvature factors on the level factor, from a non-zero value during the normal regime to a zero vector during the ZLB regime induces two kinds of changes in the dynamics of the model across the two regimes. First, it affects the dynamics of the market price of risk, which can broadly be understood as representing investors' degree of aversion to systematic macroeconomic risk. Specifically, the market price of risk becomes much more responsive to changes in the slope and curvature factors during the lower bound

regime, whereas it mainly responds to changes in the level factor during the normal regime.

In addition, the magnitude of $g_{st}^{\mathbb{P}}$ is closely related to the unconditional variance of the level factor. As such, the sharp decline in the magnitude of $g_{st}^{\mathbb{P}}$ during the lower bound regime has the effect of further compressing the variance of the level factor and by extension the volatility of short-term yields when the lower bound is binding.

Taken together, the change in $g_{st}^{\mathbb{P}}$ across the normal and lower bound regimes both reduces the conditional variance of the level factor, and amplifies the influence of the slope and curvature factors on the market prices of risk. This suggests a connection between the compression of yields and investors' perception of risk. When the lower bound is binding, short-term yields are trapped at a level close to zero, rendering them less informative about monetary policy. For investors, this equates to losing a significant source of information. As a result, they turn to alternative indicators, such as the yield spread, to guide their risk assessments. In our model, yield spreads are calculated based on the slope and curvature factors, as shown in the appendix. Therefore, the influence of these two factors on the market price of risk is naturally stronger in the lower bound regime. This implies that the market prices of risk become more volatile during periods when the ZLB is binding.

Since the TP is the additional return that investors require as compensation for the risk associated with holding longer-term yields, changes in the volatility of the market price of risk leads to corresponding changes in TP. Table 3 supports this by showing how the factors affect the TP in both the FS-ZLB model and the FS-only model. Notably, the slope factor loads significantly on the TP during the lower bound regime for yields of all maturities. The curvature factor, meanwhile, loads heavily on yields of lower maturities but its influence peters out when it comes to yields of higher maturities during the lower bound regime. This reflects the fact that the curvature factor holds information about middle term yields, while the slope factor is primarily concerned with long term yields. The active role the slope and curvature factors play in determining the TP in the FS-ZLB model provides a

stark contrast to the FS-only model, where the loadings of the two factors on the TP are rarely statistically significant.

5 Conclusion

In this paper, we investigate the effect that ZLB-induced structural changes have on term premium estimation in term structure models with falling stars, in light of the overwhelming evidence supporting the existence of structural shifts in the short end of the yield curve that arise when the lower bound is binding. The investigation is carried out using a workhorse regime-shifting AFNS model that embeds these kinds of structural changes into the lower bound regime, and also includes falling stars, in the form of a level factor that is non-stationary during the normal regime.

The results indicate that our model identifies observations from both the GFC and the COVID-19 pandemic as part of the lower bound regime, demonstrating its capability to sensibly identify the lower bound regime. Additionally, our model can replicate patterns of yield volatility observed in other studies on the ZLB, which confirms the effectiveness of our Markov switching framework in accounting for the lower bound. Furthermore, our model provides better short rate forecasts for horizons of three, six and twelve months compared to both a standard random walk specification and a model with only falling stars. The improvement in forecasting ability is especially marked during the lower bound regime. Taken together, these results show how our model outperforms standard models of the term structure of interest rates.

The TP estimate in our model exhibits significantly greater stationarity than estimates of the term premium obtained from traditional Gaussian ATSMs, which is often the case for ATSMs with falling stars. We observe in addition that Gaussian ATSMs underestimate the TP throughout the sample period. This occurs because the expected component of the

term premium tends to be overestimated in Gaussian ATSMs, as the short rate in these models is stationary and thus tends to quickly revert to a highly positive long-run mean.

We also note that, in comparison with the FS-ZLB model, a model that accounts for only falling stars underestimates TP volatility during the lower bound regime and the level of the TP during the normal regime. The distinct behavior of the term premium in the FS-only model compared to the FS-ZLB model when the lower bound is binding can be traced back to how the slope factor influences the TP. Our findings suggest that the slope factor becomes a crucial driver of the term premium and market prices of risk when the lower bound is in effect, for which a plausible explanation is the loss of the short-term interest rate as a source of information when yields are compressed. From this perspective, the TP obtained from a model that considers only falling stars falls short in capturing the high volatility of the TP over the period where the lower bound is binding, because it fails to account for the way the ZLB, through structural changes in yield volatility, alters economic agents' perception of macroeconomic risk.

The differences in TP estimates during the normal regime also admit an interesting interpretation. In the FS-ZLB model, the cumulative probability of entering into the lower bound regime grows larger with forecast horizon, which exerts downward pressure onto short rate expectations. Even apart from the direct effect that the compression of volatility has on the market price of risk during lower bound episodes, the very fact that investors are aware of potential structural changes indirectly affects TP estimation across the entire sample period.

The empirical exercise conducted in this paper thus reveals that structural changes in the dynamics of the term structure has far-reaching and surprising implications for the TP. By altering how agents perceive risk, a binding ZLB amplifies the volatility of the TP; and by changing agents' expectations of future short rates, the presence of these kinds of structural changes raises the level of the TP even during the normal regime. In a world

that has suffered through two economic crises accompanied by a binding interest rate lower bound, these findings demonstrate that the ZLB and the related structural changes will continue to exert a huge influence over the TP going forward.

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